

Harmonic maps between complete noncompact surfaces

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Abstract

In early 1990's, Schoen made two conjectures on harmonic diffeomorphisms between complete noncompact surfaces with canonical metric. In particular, the Dirichlet problem for quasiconformal harmonic diffeomorphisms from Poincaré disc onto itself and the non-existence of harmonic diffeomorphism from the complex plane on the Poincaré disc are central problems in the study of the conjectures. We are going to review the progress on these conjectures with emphasis on exponential growth open harmonic embedding of the complex plane into the Poincaré disc. Our results showed that there might be an interesting relationship between the distribution of the ideal boundary points of the image under such a harmonic map and the distribution of zeroes of the entire function given by the corresponding Hopf differential.

As examples for the relationship, we consider two cases. (1) It was proved few years ago by the speaker and others that if the Hopf differential is given by a polynomial of degree d , then the image of the corresponding open harmonic embedding touches the ideal boundary of the Poincaré disc at $d+2$ points. The converse is also true. (2) More recently, we can show that if the Hopf differential is given by an entire function of the form $P \exp(Q)$, where P and Q are polynomials such that the degree of Q is equal to k , then the image touches the ideal boundary at countable infinitely many points with exactly k accumulation points.