Finite time blow-up of 3 harmonic map heat flow

Leungfu Cheung

Abstract

In this talk, we report on our recent result concerning finite time blow-up for solutions of the 3-harmonic map heat flow equation from a ball in the 3-dimensional Euclidean space to the 3-sphere.

The 2-harmonic map heat flow equation is widely studied and it is well-known that when the target is negatively curved, then solution exists for all time. In the case when the target is positively curved, blow-up phenomenon may occur. However, when the dimension of the domain is two, it had been believed that no blow-up occurs. In 1992, Chang, Ding and Ye showed that this is in fact not true by giving initial data for which the solutions to the harmonic map heat flow from the disc to the 2-sphere blow up.

In our study, we attempt to show that the blow-up phenomenon is in fact related to the conformality of the underlying energy functional. In short, we conjecture that blowup occurs for p-harmonic map heat flow into spheres when p equals the dimension of the domain.

Concerning the above conjecture, we have to date verified the blow-up phenomenon in the case p=n=3. We proved this by extending the comparison techniques of Chang, Ding and Ye to degenerate parabolic systems. As a candidate for the comparison function, we use solutions of the related degenerate ordinary differential equations, which we studied in our previous papers on rotationally symmetric p-harmonic maps.